Network Analysis Workshop
ISEM 2023, Toronto, Canada

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Agenda

1) Introduction to Networks and ENA/NEA (Fath)
2) Network Construction
3) Network Analysis in R: enaR (Borrett)
    1) Introduction
    2) Guided exercises to use the software
        1) Exercise 1: Getting Started with enaR (use script to guide)
        2) Exercise 2: "micro-modeling" – fictious 3-node network
        3) Exercise 3: Comparing Models & Batch Processing
        4) Exercise 4: Uncertainty analysis (if time allows).
4) Network Applications (Fath)

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Outline

1) The modelling problem
   1) from nature → concept → formalism
2) Systems Theory
3) Network Analysis
   i. Structural Paths

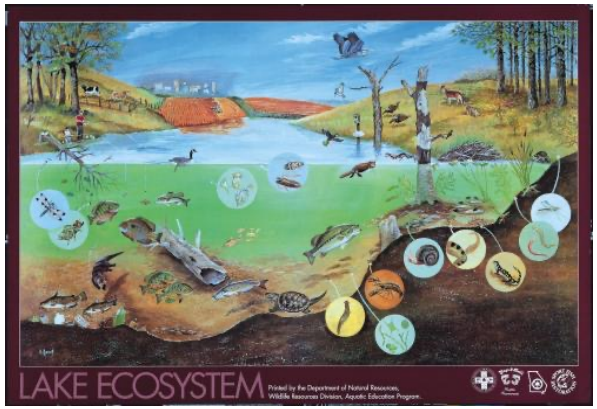
3

The Modelling Problem

From nature to conceptual model: many assumptions, abstractions, simplifications

4

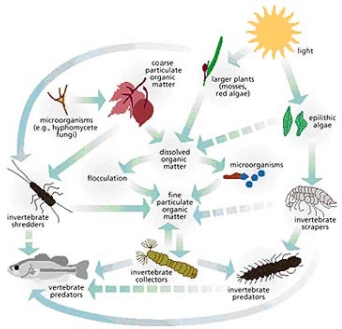
Identifying compartments



Sampling I: What is the abundance and distribution of species?

5

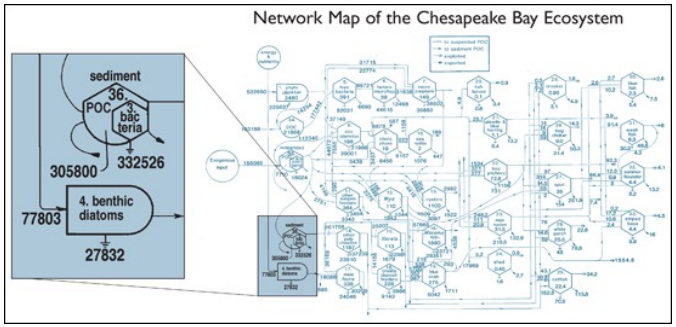
Identifying connections



Sampling II: How are they connected qualitatively?

6

Identifying the controls:



Sampling III: how much is flowing between compartments?

Quantifying the network empirically or through simulation

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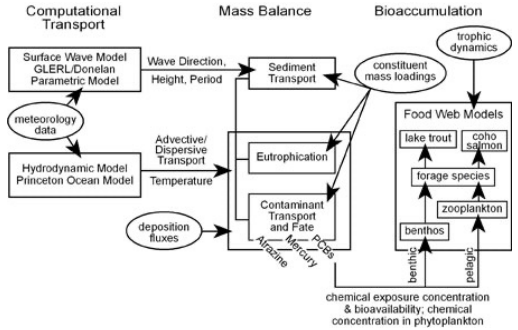
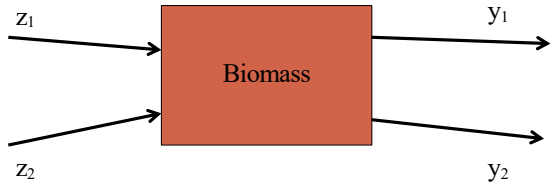


Adding other influencing (environmental) factors

8

From conceptual models (pictures) to equations
in principle this is an isomorphic mapping

Each compartment (state variable) becomes one equation



$$\frac{\Delta B}{\Delta t} = \text{input} - \text{output}$$

$$\frac{\Delta B}{\Delta t} = (z_1+z_2) - (y_1+y_2)$$

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Pelagic

Ecosystem Model for Rawal Lake

List of counts

$$\dot{x_1} = \frac{dx_1}{dt} = f_{0} + \mu_{1,0}x_1 - f_{6,1}x_1 - f_{11,1}x_1 - f_{2,1}x_1 - f_{3,1}x_1 - \lambda_{0,1}x_1 \quad (1)$$

$$\dot{x_2} = \frac{dx_2}{dt} = f_{2,1}x_1 + f_{2,6}x_6 - f_{3,2}x_2 - f_{4,2}x_2 - f_{8,2}x_2 - f_{11,2}x_2 - \lambda_{0,2}x_2 \quad (2)$$

$$\dot{x_3} = \frac{dx_3}{dt} = f_{3,1}x_1 + f_{3,2}x_2 - f_{3,6}x_6 - f_{3,7}x_7 - f_{4,3}x_3 - f_{10,3}x_3 - f_{11,3}x_3 - \lambda_{0,3}x_3 \quad (3)$$

$$\dot{x_4} = \frac{dx_4}{dt} = f_{4,3}x_3 + f_{4,2}x_2 + f_{4,7}x_7 - f_{11,4}x_4 - \alpha_{0,4}x_4 - f_{4} - \lambda_{0,4}x_4 \quad (4)$$

$$\dot{x_5} = \frac{dx_5}{dt} = f_{5} + \mu_{5,0}x_5 - f_{7,5}x_5 + f_{9,5}x_5 - f_{10,5}x_5 - f_{6,5}x_5 - f_{11,5}x_5 - \lambda_{0,5}x_5 \quad (5)$$

$$\dot{x_6} = \frac{dx_6}{dt} = f_{6} + \mu_{6,0}x_6 + f_{6,1}x_1 + f_{6,5}x_5 - f_{2,6}x_6 - f_{3,6}x_6 - f_{11,6}x_6 \quad (6)$$

$$\dot{x_7} = \frac{dx_7}{dt} = f_{7,5}x_5 + f_{7,11}x_{11} - f_{3,7}x_7 - f_{4,7}x_7 - f_{8,7}x_7 - f_{11,7}x_7 - \lambda_{0,7}x_7 \quad (7)$$

$$\dot{x_8} = \frac{dx_8}{dt} = f_{8,10}x_{10} + f_{8,7}x_7 - f_{11,8}x_8 - \alpha_{0,8}x_8 - \lambda_{0,8}x_8 \quad (8)$$

$$\dot{x_9} = \frac{dx_9}{dt} = f_{9,5}x_5 - \alpha_{0,9}x_9 - \lambda_{0,9}x_9 \quad (9)$$

$$\dot{x_{10}} = \frac{dx_{10}}{dt} = f_{10,3}x_3 + f_{10,5}x_5 - \alpha_{0,10}x_{10} - \lambda_{0,10}x_{10} \quad (10)$$

$$\dot{x_{11}} = \frac{dx_{11}}{dt} = f_{11,3}x_3 + f_{11,1}x_1 + f_{11,2}x_2 - f_{11,7}x_7 - f_{7,11}x_{11} - \lambda_{0,11}x_{11} \quad (11)$$

Figure 2: Block diagram of conceptual Rawal Lake ecosystem network model, the different colors represent availability of biomass data for each compartment, arrow lines show the flow of energy in the lake ecosystem

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Review: 4 Cs of modelling

- Compartments (X) = state variable [amount], measures a state of the system
- Connections (F) = flows of energy or matter [amount per time]
- Control (c) = parameter or function that controls the flow [(often) per time]
- Currency = the units of the model (energy, $, carbon, etc.)

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Qualitative conceptual models

- Identifying the boundary
- Defining the compartments/state variables
- Identify positive (reinforcing) and negative (balancing) feedbacks

- System structure: topology of transactions
- System function: quantification of transactions

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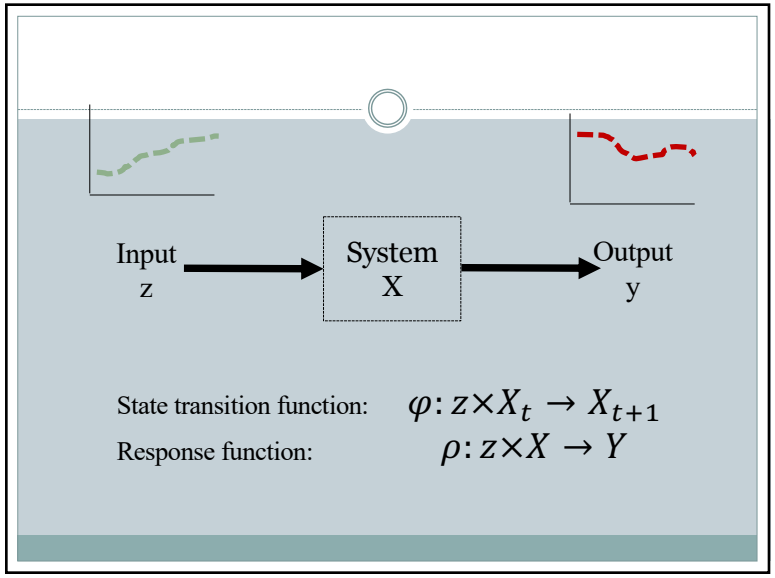

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Quantitative Mathematical models

- Generalization/abstraction of nature into a formal system
- Model contains hypotheses about how the system functions
- Gaps in data
- Gaps in understanding
- Sensitivity analysis
- Scenario analysis – "what if"
- Inverse model – how to reach a desired state
- Indirect linkages

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STELLA software (www.iseesystems.com/store/products/stella-architect.aspx)
Vensim software (https://vensim.com/)

- Dynamical systems simulation packages
- You draw the model structure and put in the initial conditions and parameters, and it solves the system of differential equations to generate the output

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Why bother with modelling and systems approaches?

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Today's problems largely stem from the solutions to yesterday's problems.

Lack of systems perspective guarantees *unintended side effects*

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Systems Theory

- "...is, strictly speaking, not a theory of systems, but of system-environment distinctions." *On Luhmann* Moeller *2006, p. 40*

"If you want to make an apple pie from scratch, first you must make a universe." -- Carl Sagan

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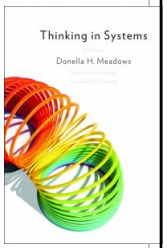
why networks are important

Analyzing the network can help avoid unwanted or unexpected consequences.



If everything is connected to everything else, then how can we ever know anything?

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2) Network analysis

- Represent systems as interacting components
- Used to identify and quantify direct and indirect effects in the system
- Many kinds of networks

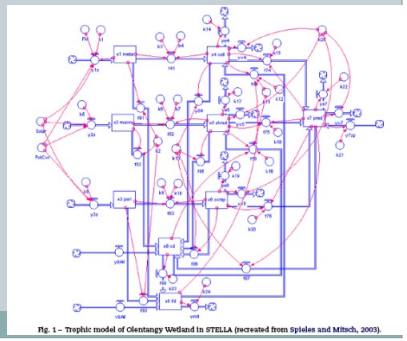
Fig. 1 – Trophic model of Olentangy Wetland in STELLA (recreated from Spieles and Mitsch, 2003).

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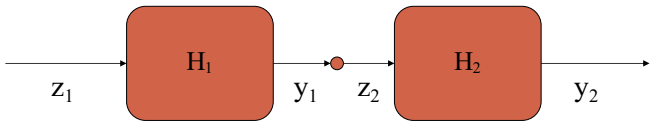
Systems (networks) arise from the bonds between entities



$y_1 \equiv z_2$

Identity constraint of coupling

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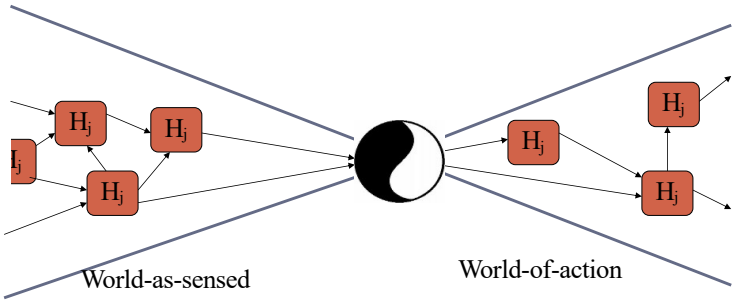
Proposition 1 (Patten 1978):

Every object defines two environments



Input and Output Duality

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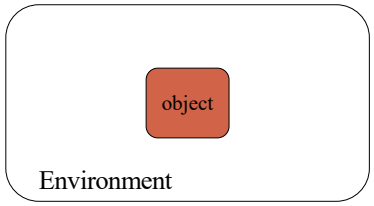
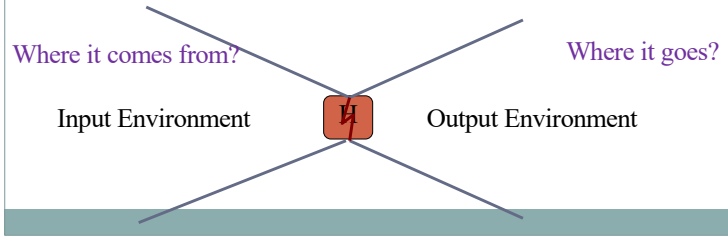
Old perspective, fragmentation between object and environment



New perspective, object is focus of two environments



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Environ duality

- "The difference system/environment occurs twice: as the difference *produced by* the system, and as the difference *observed within* the system." *On Luhmann p. 68*
- "Every object H defines two environments: an input environment and on output environment" *Patten, 1978*

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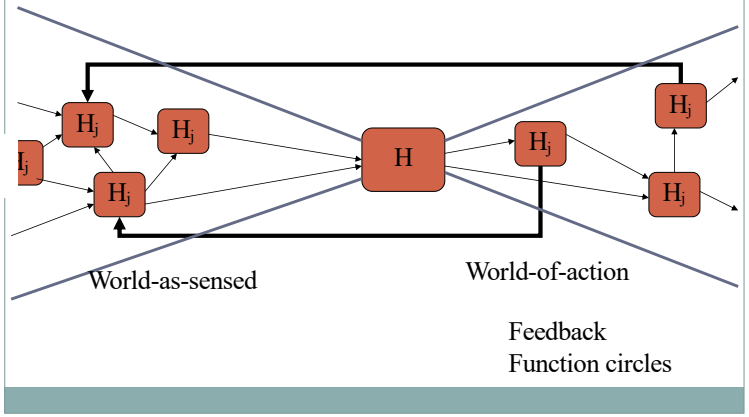
Proposition 1:

Every object defines two environments



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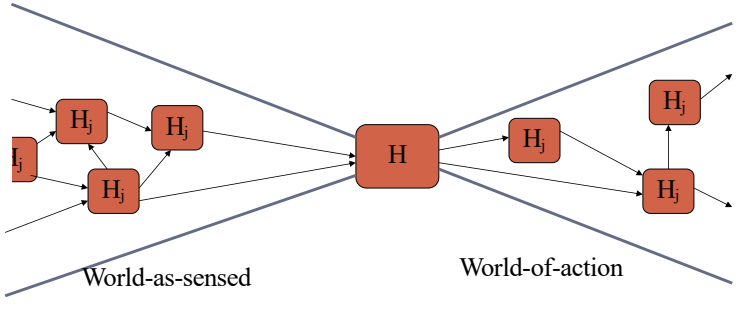
Proposition 1:

Every object defines two environments



"Infinite regress"

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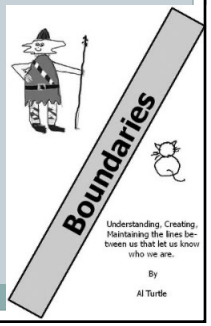
Boundaries and otherness

- "The prerogative of realization of internal system structure is that of environment." *Patten 1978, p.211*
- "By constructing itself as a system, a system also constructs its understanding of the environment." *Moeller 2006, p. 16*



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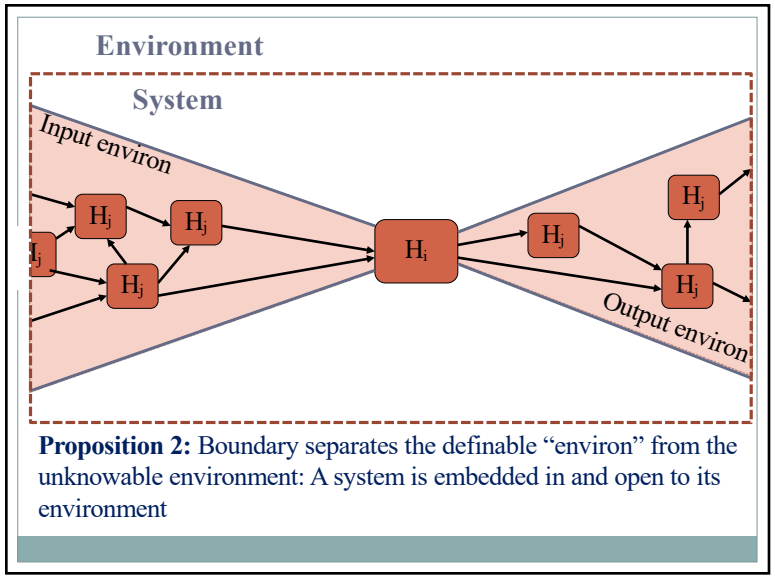

Proposition 2: Boundary separates the definable "environ" from the unknowable environment: A system is embedded in and open to its environment

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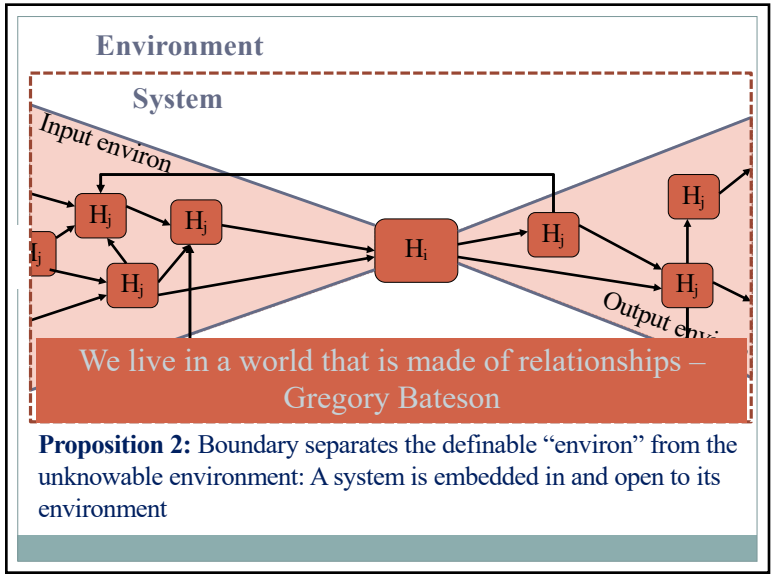

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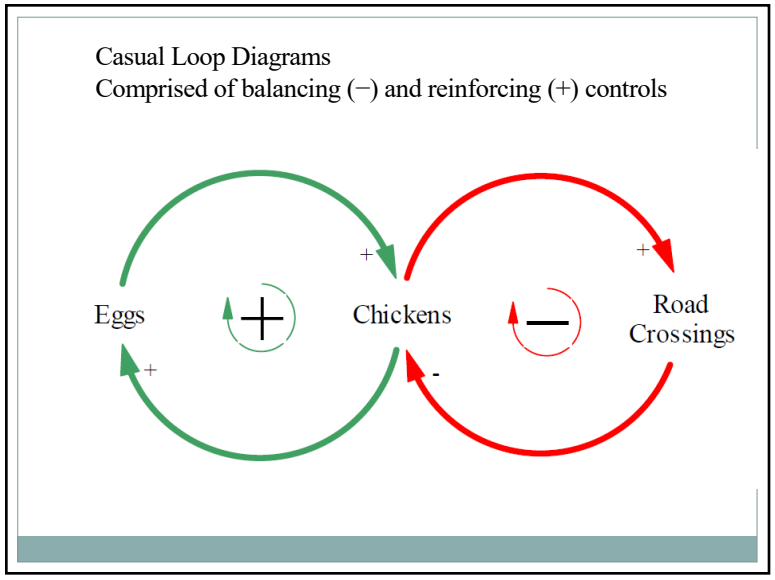

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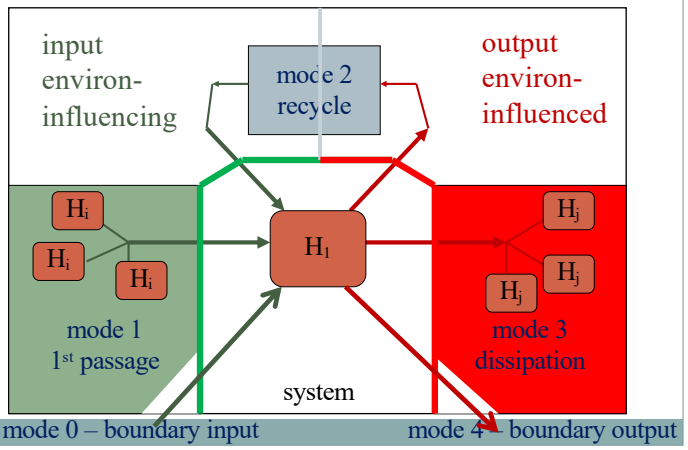
Proposition 3: Environs form a partition of the system.



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Many different kinds of networks

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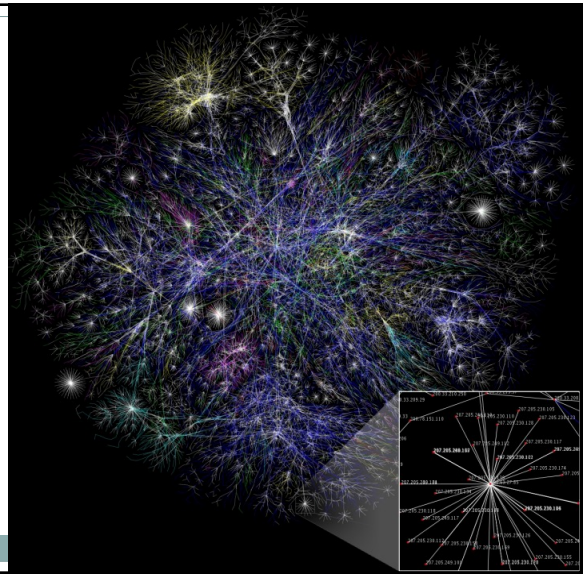
The Internet in 2005



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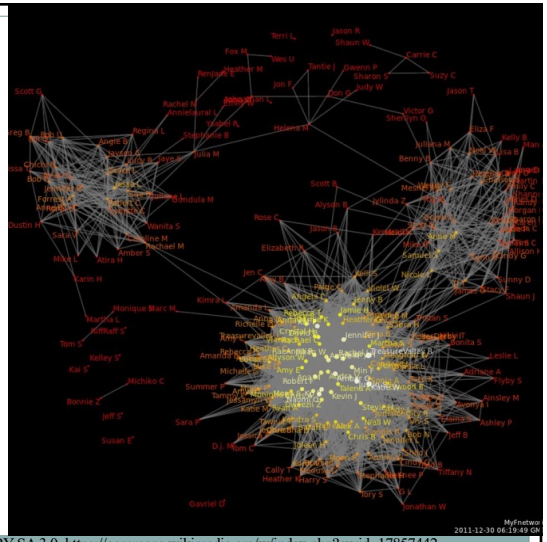
Friendship ties on Facebook

Strength of weak ties (Granovetter 1973)



By Kencf0618 - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=17857442

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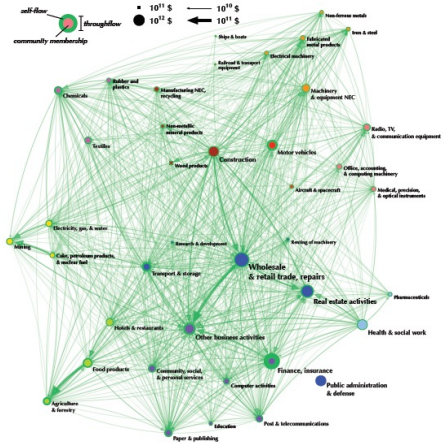
Economic Network – flows of money between industrial sectors of USA



McNerney et al., 2013

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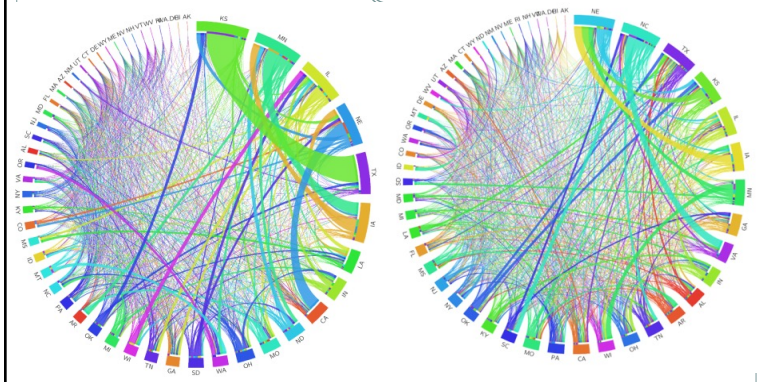
Cereal and meat trade in US



Cereals Meat

Figure by Nemi Vora

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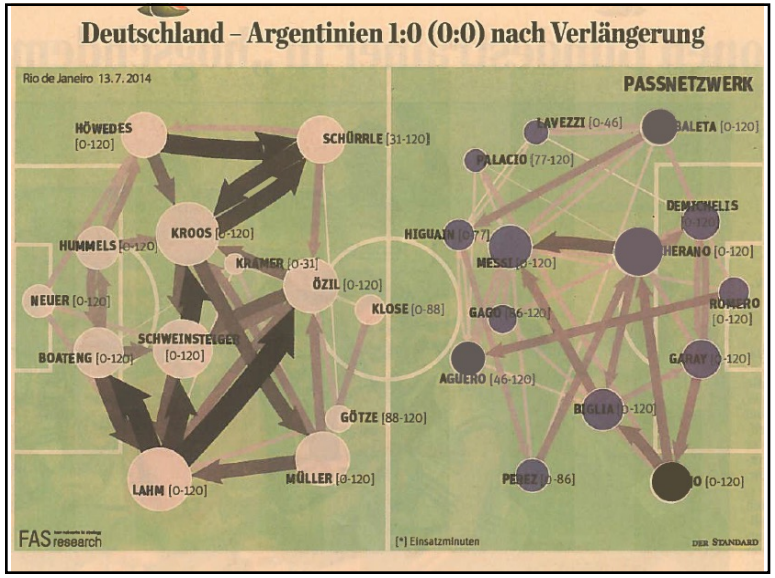
Deutschland – Argentinien 1:0 (0:0) nach Verlängerung
Rio de Janeiro 13.7.2014
PASSNETZWERK
HÖWEDES [0-120]
SCHÜRRLE [31-120]
KROOS [0-120]
HUMMELS [0-120]
KRAMER [0-31]
ÖZIL [0-120]
NEUER [0-120]
KLOSE [0-88]
SCHWEINSTEIGER [0-120]
BOATENG [0-120]
GÖTZE [88-120]
LAHM [0-120]
MÜLLER [0-120]
LAVEZZI [0-46]
PALACIO [77-120]
HIGUAIN [0-77]
MESSI [0-120]
GAGO [46-120]
AGUERO [46-120]
PEREZ [0-86]
FAS research
[*] Einsatzminuten
DER STANDARD

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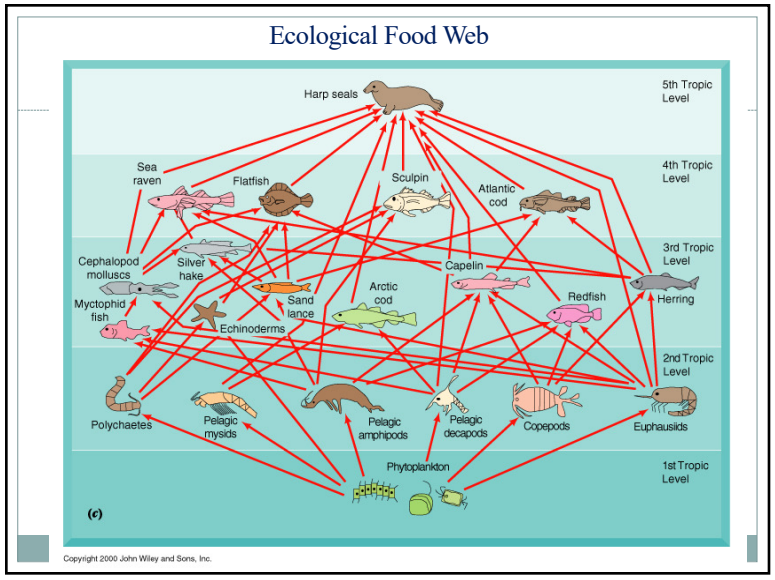
Ecological Food Web
5th Tropic Level
4th Tropic Level
3rd Tropic Level
2nd Tropic Level
1st Tropic Level
Harp seals
Sea raven
Flatfish
Sculpin
Atlantic cod
Cephalopod molluscs
Silver hake
Capelin
Myctophid fish
Sand lance
Arctic cod
Redfish
Herring
Echinoderms
Polychaetes
Pelagic mysids
Pelagic amphipods
Pelagic decapods
Copepods
Euphausiids
Phytoplankton
(c)
Copyright 2000 John Wiley and Sons, Inc.

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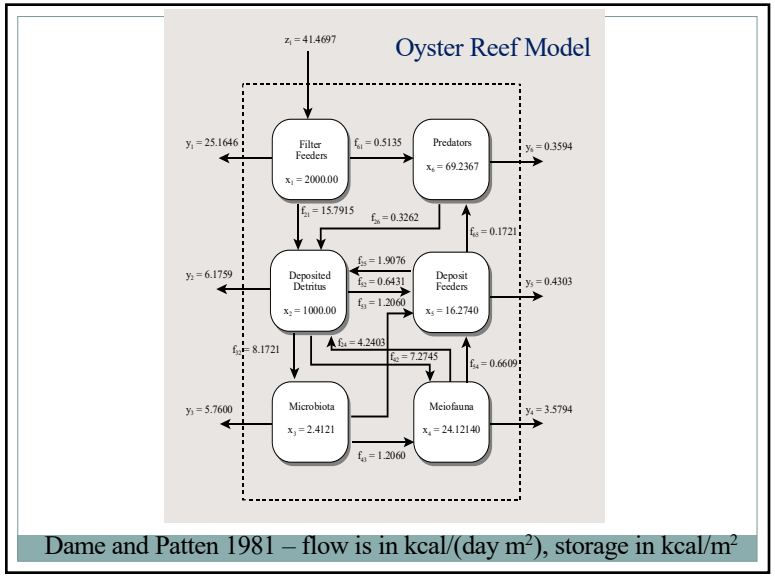

Dame and Patten 1981 – flow is in kcal/(day m^2), storage in kcal/m^2

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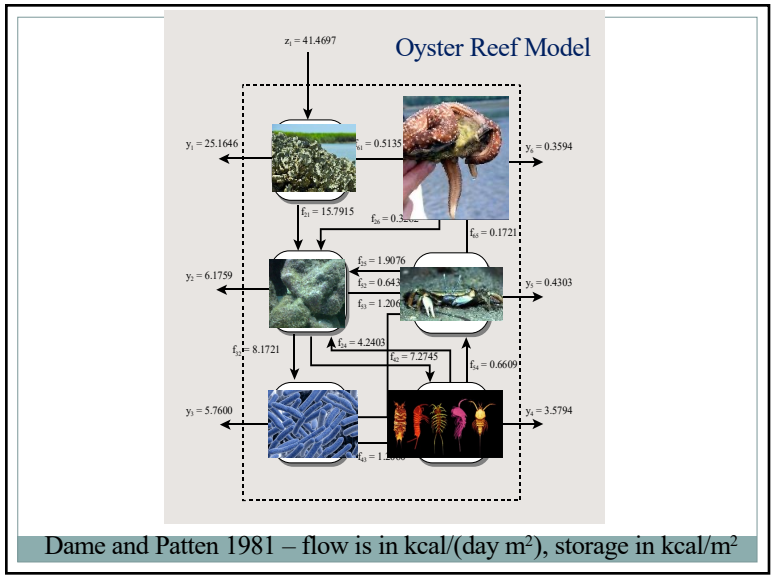

Dame and Patten 1981 – flow is in kcal/(day m^2), storage in kcal/m^2

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Back to building models

1. Pick a system of interest to you
2. Draw the model
3. Identify all the parts/actors
4. Identify if they are connected
5. What are the relations between them? Balancing or reinforcing?

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Ecological Network Analysis

Input-Output based approach

Path Analysis – a_{ij} – enumerates number of pathways in a network

Flow Analysis ($g_{ij} = f_{ij}/T_j$) – identifies flow intensities along indirect pathways

Utility Analysis ($d_{ij} = (f_{ij} – f_{ji})/T_i$) – identifies utility intensities along indirect pathways

Information based approach

Ascendancy– AMI – average mutual information

Robustness – trade-off of efficiency and redundancy

Pointwise mutual information – pairwise transaction assessment

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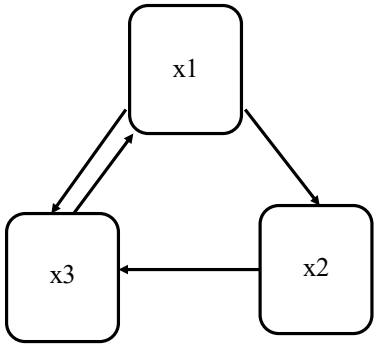
How to measure structure (topology) and indirectness

Example – digraph to adjacency matrix



$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

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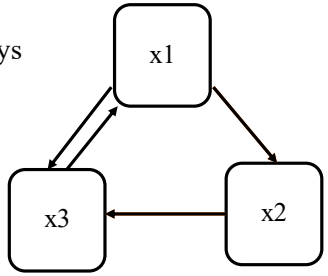
Matrix multiplication gives
Higher Order (Indirect) Pathways

A^m, where $m > 1$



$$A^2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad A^4 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad A^5 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

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Powers of a matrix!!

The matrix A^m gives exactly the number of walks between two nodes of length m.

$\mathbf{A}^1$ are the direct walks.
$\mathbf{A}^2$ are the walks that take two steps
$\mathbf{A}^3$ are the walks that take three steps, etc.

Notice that some elements which were zero originally get filled in.

In other words we have a way to identify the indirect, i.e., m>1, walks in the matrix, and hence in the graph.

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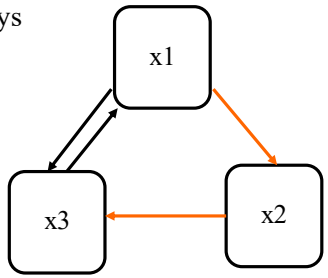
Higher Order (Indirect) Pathways

A^m, where m > 1

What happens to a_{ij} as m → ∞ ?



$$A^2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad A^3 = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix} \quad A^5 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

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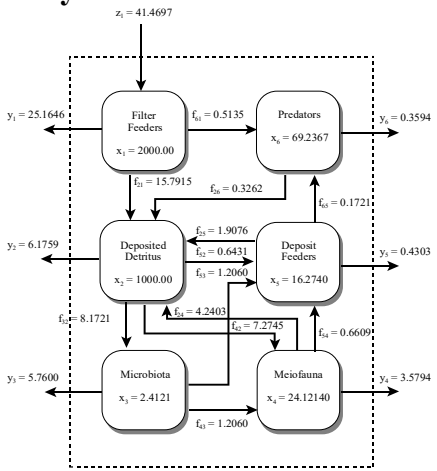
Oyster Reef Model



$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Flows are in kcal m^{-2} day^{-1}; storages are kcal m^{-2}. Dame, R. F., and B. C. Patten. 1981. Analysis of energy flows in an intertidal oyster reef. Marine Ecology Progress Series 5:115-124.

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2nd order paths

$$A^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

4th order paths

$$A^4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 7 & 5 & 6 & 6 & 4 \\ 2 & 4 & 2 & 3 & 2 & 2 \\ 3 & 6 & 4 & 4 & 3 & 2 \\ 5 & 8 & 6 & 6 & 5 & 3 \\ 3 & 3 & 2 & 3 & 3 & 2 \end{bmatrix}$$

3rd order paths

$$A^3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 4 & 2 & 3 & 2 & 2 \\ 1 & 2 & 2 & 1 & 1 & 0 \\ 2 & 2 & 2 & 2 & 2 & 1 \\ 3 & 3 & 2 & 3 & 3 & 2 \\ 1 & 2 & 1 & 1 & 1 & 1 \end{bmatrix}$$

5th order paths

$$A^5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 11 & 17 & 12 & 13 & 11 & 7 \\ 6 & 7 & 5 & 6 & 6 & 4 \\ 8 & 11 & 7 & 9 & 8 & 6 \\ 11 & 17 & 11 & 13 & 11 & 8 \\ 5 & 8 & 6 & 6 & 5 & 3 \end{bmatrix}$$

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10^{th} order paths

$$A^{10} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 519 & 759 & 518 & 595 & 519 & 354 \\ 241 & 354 & 241 & 277 & 241 & 165 \\ 353 & 519 & 354 & 406 & 353 & 241 \\ 518 & 760 & 519 & 595 & 518 & 353 \\ 242 & 353 & 241 & 277 & 242 & 165 \end{bmatrix}$$

20^{th} order paths

$$A^{20} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1083304 & 1587660 & 1083305 & 1243524 & 1083304 & 739168 \\ 504356 & 739168 & 504355 & 578949 & 504356 & 344136 \\ 739169 & 1083304 & 739168 & 848491 & 739169 & 504356 \\ 1083304 & 1587660 & 1083304 & 1243524 & 1083304 & 739169 \\ 504355 & 739169 & 504356 & 578949 & 504355 & 344135 \end{bmatrix}$$

MANY PATHWAYS FOR ENERGY/INFLUENCE TO TRAVEL!

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Number of paths of length m from compartment 1 to compartment i for the Oyster Reef Model.

m	1→2	1→3	1→4	1→5	1→6
1	1	0	0	0	1
2	1	1	1	1	0
3	2	1	2	3	1
4	6	2	3	5	3
5	11	6	8	11	5
6	24	11	17	25	11
7	53	24	35	52	25
8	113	53	77	113	52
9	241	113	165	242	113
10	519	241	353	518	242
11	1113	519	760	1113	518
12	2391	1113	1632	2392	1113
13	5137	2391	3504	5136	2392
14	11032	5137	7528	11032	5136
15	23696	11032	16169	23696	11032
16	50898	23696	34728	50897	23696
17	109322	50898	74594	109322	50897
18	234813	109322	160220	234814	109322
19	504356	234813	344135	504355	234814
20	1083304	504356	739169	1083304	504355
$a_{ij}^{(20)}/a_{ij}^{(19)}$	2.147896	2.147905	2.147904	2.147900	2.147892

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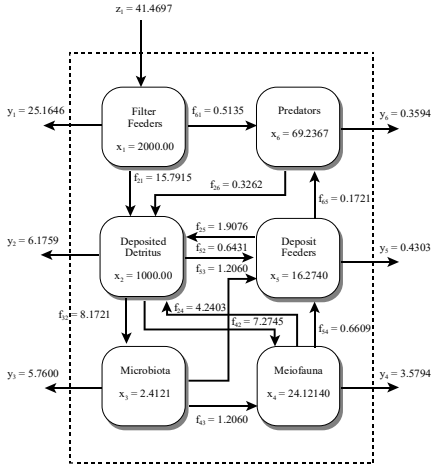
Oyster Reef Model



$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\lambda_{\max} = 2.15$$

$$\lim_{m \to \infty} \frac{a_{ij}^{(m+1)}}{a_{ij}^{(m)}} = \lambda_{\max}$$

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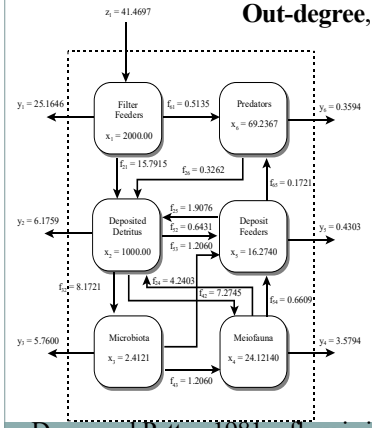
In ecological food webs

In-degree, $A_{i.}$, measures the food sources

Out-degree, A_{j}, measures the # of predators



out

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

in

0 4 1 2 3 2

2 3 2 2 2 1

Adjacency matrix

Dame and Patten 1981 – flow is in kcal/(day m^2), storage in kcal/m^2

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Back to building models

1. Re-examine the network you've drawn
2. Re-examine the relations between them?
3. Construct an adjacency matrix of the network
4. Analyze the path structure and indirect pathways

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Network Environ Analysis

Path Analysis – a_{ij} – enumerates number of pathways in a network

Flow Analysis (/T_j) – identif s along

More to follow . . .

Storage Analysis ($c_{ij} = f_{ij}/x_j$) – identifies storage intensities along indirect pathways

Utility Analysis ($d_{ij} = (f_{ij} - f_{ji})/T_i$) – identifies utility intensities along indirect pathways

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THANK YOU FOR YOUR ATTENTION

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